Engineering Notes

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A Slanted Round Jet at Low Forward Speed

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Experiment

THE jet wind-tunnel model (Fig. 1) consisted of a 3-in. i.d. nozzle bolted to a plenum chamber. A flexible hose lead from the plenum chamber to an L-shaped combined air-supply and mounting tube, attached to a force balance located below the floor of the 7-ft × 10-ft test section of the tunnel. This tube was enclosed in a nonmetric streamlined windshield bolted to the tunnel floor. The nozzle and the plenum chamber could be rotated in pitch to vary the jet angle of attack α. The nozzle/plenum/hose assembly was enclosed in a metric oblate ellipsoid fairing (Fig. 2) of fineness ratio 2.4 (thickness $12\frac{1}{4}$ in., diameter $29\frac{1}{2}$ in.). The model was mounted below the centerline of the tunnel to allow space for jet turning. The highest point on the ellipsoid was located on the centerline of the test section. This gave a 1-ft. clearance between the tunnel floor and the lowest point on the ellipsoid. The jet was mounted upside down as compared to, for instance, a lift engine in a V/STOL aircraft. Lift is therefore positive in the downward direction.

The model was bench tested, and the jet exit velocity was determined to be uniform to within 3%. In the tunnel, measurements were made of lift L, jet-stream total-head difference ΔH, jet and freestream temperatures, and jet mass flow rate $\dot{m}_j$. The tests were run at constant mass flow (3.13 lb/sec, approximately). Prior to each run at $V_\infty/V_{j\infty} \simeq$ 0.1, 0.2, 0.3, and 0.4 with the jet velocity $V_{j\infty} \simeq 750$ fps (corresponding to ambient pressure), jet-off runs with the exit plugged were made. Tunnel-air-on lift tares were recorded and are plotted in Fig. 3. The tares are positive, indicating a force tending to pull the model towards the floor. These lift tares have been subtracted from the measured jet lifts presented in Fig. 4. As shown, a distinct decrease in jet lift with increasing forward speed ratio at constant angle of attack and mass flow was recorded for $\alpha = 21.6°$ and $44.0°$. Tests were also run at $\alpha \simeq 90°$. The data have been omitted, however, because of unsatisfactory (nonlinear) lift tare variation and unknown wall interference effects.

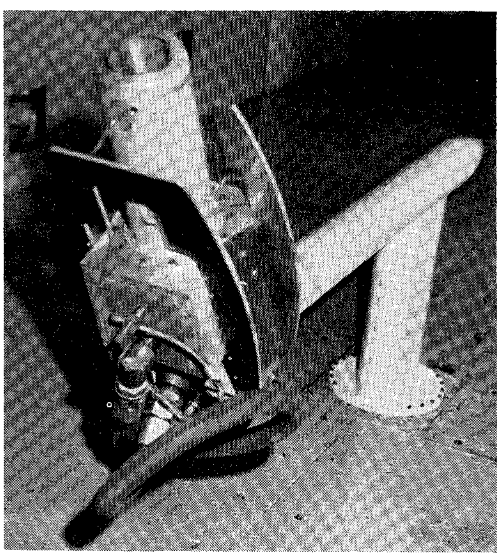

Fig. 1 Jet model installed in LTV wind tunnel (ellipsoid fairing removed, $\alpha \simeq 120°$).

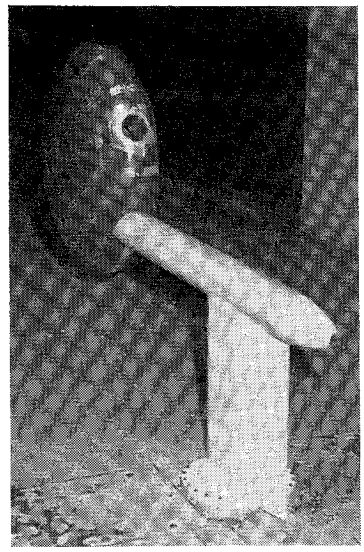

Fig. 2 Jet model installed in LTV wind tunnel (ellipsoid fairing in place, α = 21.6°, tunnel flow from upper left to lower right corner).

Theory

A large number of investigators have in the past experimentally determined the flow phenomena associated with a jet in a cross flow. From their reports, as well as from smoke observations in the present tests, it is clear that at low forward speeds ($V_\infty \ll V_{j\infty}$) the jet remains essentially circular and axially straight for several diameters after leaving the nozzle, before curving with the stream and eventually rolling up into two counter-rotating concentrated vortices, which are then slowly dissipated by viscous action (Fig. 5).

Analysis of the problem of a round jet in a cross flow has in the past mostly been limited to determination of the jet centerline by semiempirical means. An exception to this depressing state of the art is given by the inviscid-flow Trefftz-plane solution of Ref. 1. This closed form solution is exact insofar as the boundary conditions of flow tangency and

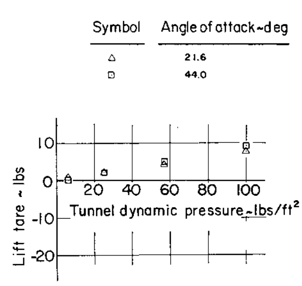

Fig. 3 Measured lift tares, jet flow off, exit plugged.

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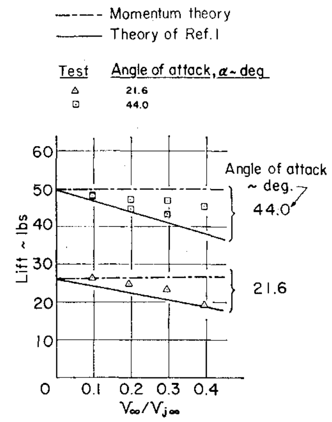

Fig. 4 Comparison between calculated and measured lift at approximately constant mass flow (calculation based upon measured ΔH and mass flow).

equality of pressure on either side of the interface are satisfied without approximations, assuming no roll up. By a conventional momentum-flux control-surface method the normal and axial force components are in Ref. 1 shown to be, respectively,

$$Z = \rho A_{j\infty}(1 - \mu)u_\infty w_\infty \tag{1}$$

$$X = -\rho A_{j\infty}\left\{u_j(u_j - u_\infty) + \frac{1-\mu}{\mu^2}w_\infty{}^2 \times \left[(1+\mu^2)\left(1 - \frac{1-\mu}{4\mu}\ln\frac{1+\mu}{1-\mu}\right) - \frac{1-\mu}{2}\right]\right\} \tag{2}$$

The velocity components u_∞, w_∞, and u_j are defined in Fig. 5. Resolving the resultant force into components normal to and along the freestream direction, the lift and drag of a slanted jet engine in a stream become

$$L = \dot{m}_j(u_j - V_\infty \cos\alpha)\sin\alpha + [2(1-\mu)\sin\alpha - 2(C-\mu)\sin^3\alpha]q_\infty A_{j\infty} \tag{3}$$

$$D = -\dot{m}_j(u_j - V_\infty\cos\alpha)\cos\alpha + [2(C-\mu)\sin^2\alpha\cos\alpha]q_\infty A_{j\infty} \tag{4}$$

where q_∞ is the freestream dynamic pressure, and

$$\mu = (V_\infty/V_{j\infty})\cos\alpha/[1 - (V_\infty/V_{j\infty})^2\sin^2\alpha]^{1/2}$$
$$u_j = [2(\Delta H + q_\infty\cos^2\alpha)/\rho]^{1/2}$$
$$A_{j\infty} = \dot{m}_j/\rho u_j$$
$$C = [1 + (1/\mu^2)]\{\mu - \tfrac{1}{2} + [(1-\mu)^2/4\mu] \times \ln[(1+\mu)/(1-\mu)]\}$$

In dealing with wind-tunnel models having their own external air supply for the jet, a term $\dot{m}_jV_\infty$ (the inlet drag) must be subtracted from Eq. (4). In writing Eqs. (3) and (4), the tacit assumption has been made that the angle of inclination of the jet in the Trefftz plane is the same as the jet exit angle α. This should be valid when $V_\infty \ll V_{j\infty}$.

Equation (3) is plotted on Fig. 4 for comparison with the measured data. It is apparent that the theory of Ref. 1 exhibits the correct trend of lift decrease with increasing forward speed at constant mass flow and angle of attack. Also shown in Fig. 4 for comparison are the constant values of jet lift ($L = \dot{m}_jV_{j\infty}\sin\alpha$) calculated by momentum theory,

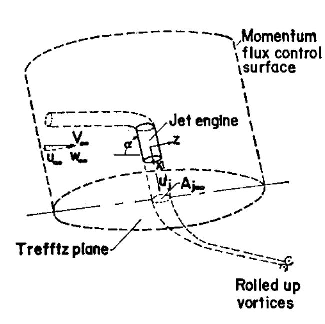

Fig. 5 Flow model and coordinate system.

which, of course, does not take into account the trailing vortex system on the interface. It is concluded that the new theory may be of value for improved estimates of the variation of the lift of a slanted round jet with forward speed.

Reference

[1] Levinsky, E. S. et al., "Lifting-Surface Theory for V/STOL Aircraft in Transition and Cruise. I," *Journal of Aircraft*, Vol. 6, No. 6, Nov.–Dec. 1969.

An Explicit Formula for Additive Drag of a Supersonic Conical Inlet

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Nomenclature

M = Mach number
P = static pressure
P_t = total pressure
∞ = subscript to M, P or P_t indicating parameter is evaluated for freestream conditions, otherwise parameter is evaluated on ray (or conical surface) from cone tip to inlet lip
γ = ratio of specific heats
θ = half angle of conical surface from cone tip to lip
ϕ = direction of flow through conical surface relative to inlet axis

CHARTS of mass-flow ratio and additive-drag coefficient for supersonic inlets with a conical centerbody are presented by Mascitti[1] and Barry.[2] Mascitti used a definition of additive drag in terms of an integral of gage pressure from the freestream to the inlet lip and, therefore, computed the additive drag by numerical integration. This technique has the disadvantage of requiring a knowledge of the conical flowfield between the conical shock and the inlet lip. However, if an equivalent definition of additive drag as a difference in momentum is used, an explicit equation for the drag, which requires a knowledge of the flow only in the freestream and through the conical surface intersecting the inlet lip with apex at the tip of the conical centerbody, may be derived. This equation was used by UAC[2] in 1958 and was known at least six years earlier.

The formula for additive drag coefficient is

$$C_{D,a} = \frac{2}{\gamma M_\infty{}^2}\left[\frac{P}{P_\infty}\left\{1 + \gamma M^2\cos\phi \times \left(\frac{\sin(\theta-\phi)}{\sin\theta}\right)\right\} - 1\right] - 2\frac{w}{w_\infty}$$

where the mass-flow ratio is

$$\frac{w}{w_\infty} = \frac{\sin(\theta-\phi)}{\sin\theta}\frac{P_t}{P_{t\infty}}\frac{M}{M_\infty}\left(\frac{1 + (\gamma-1)M_\infty{}^2/2}{1 + (\gamma-1)M^2/2}\right)^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

The parameters of flow angle ϕ, static pressure P, total pressure P_t, and Mach number M on the ray from the cone tip to the inlet lip in these two equations may be obtained for any ray (lip) angle θ from published tables of supersonic conical flow, such as Ref. 3.

An approximate solution for $C_{D,a}$ and w/w_∞ is derived in Ref. 4 in order to avoid the numerical integration used in Ref. 1. However, if one uses tables of conical flow[3] to find the shock-wave angle required by the approximate solution,[4] one

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